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Neural network optimization of spontaneous breaking of supersymmetry

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Abstract Using mean field annealing approximation the computation of stationary probabilities at thermal equilibrium has been demonstrated using the method of optimization by neural network principles and choosing the energy function of a feedback neural network of Hopfield type. The results of this method have been used to a study of mechanism of spontaneous breaking of super symmetry (the symmetry between bosons and fermions)

Keywords : Optimization, supersymmetry, neural network,

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1. Introduction

Building intelligent system that can model human behavior has captured the attention of the world for years. It led to a such a technology as neural network which has generated enormous interest among computer scientists, physicist, mathematicians and medical scientists. The field of Artificial Neural Network (ANN) came into prominence mainly because of our ability to deal with natural phenomena in terms of process and constituents. These are several attractive paradigms which can be fused with current ANN technique. For example, Artificial Ant system [1], Cultural Evolution [2], DNA Computing [3] and Immunity Net [4] seem to be attractive and viable approaches that can be amalgamated with ANN. One most important advantage of ANN is that it is adaptive and hence many existing paradigms can be fused into it easily. Although, as of now there are no guidelines for developing hybrid paradigms, the urge to develop models to perform human cognition tasks will continue to motivate researches to explore new directions in the field. The most important issue for solving practical problems using the principles of ANN is still in developing a suitable architecture to solve a problem. It continues to dominate this research area. ANN research has to expand its scope to take into account the fuzzy nature of real data and reasoning. Computer scientists are recently analyzing the chaotic dynamics of feedback networks [5] by employing the periodic attractors embedded in each chaotic attractor to store input pattern [6].

One of the most prevalent uses of neural networks is neural optimization (NO) which is a technique for solving a problem by casting it into a mathematical equation that, when either maximized or minimized, solves the problem without going in to detailed dynamics of the concerned physical system. In other words, one of the most successful applications of neural network principles is in solving optimization problems [6,7]. There are many situations where a problem may be formulated as minimization or maximization of some cost function or objective function subject to certain constraints. It is possible to map such a problem to a feedback network, where the units and connection strengths are identified by comparing the cost function of the problem with the energy of the network expressed in terms of the state values of the units and the connection strengths. The neural optimization casts the optimization problem into the form of an energy function that describes the dynamics of a neural system. The neural network dynamics are such that the network always seeks a sizable state when the energy function is at a minimum, then the network will automatically find a solution. The inputs to the neural networks are the initial state of the neural networks, and the processing element (PE) values represent the parameters of a solution. It has been demonstrated [8] that how highly interconnected network of simple analog processor can collectively compute good solutions to difficult optimization problems.

Starting with the energy function of a feedback, neural network of Hopfield [9] type, in the present paper, we have determined the state of the network at the global minimum of the energy function overcoming its local minima by adopting a simulated annealing for the computation of stationary probabilities at thermal equilibrium for each temperature in the annealing schedule. We have used the mean field annealing approximation [10] to speed up the process of simulated annealing replacing the fluctuating activation values of each unit by its average values and solved the recurrence equations involving the average values of the state units by the method of iteration starting with some arbitrary initial values and lowering the temperature after the steady equilibrium. The results of this method have been applied to undertake the study of mechanisms of spontaneous breaking of supersymmetry and also the breaking to underlying gauge symmetry which are the current problems of enormous potential importance in High Energy Particle Physics because if supersymmetry is relevant to the real world, it is important to investigate the mechanism by which it could be broken

2. Optimization by neural network principles

This problem involves the determination of the state of network at the global minimum of the energy function. In this process, it is necessary to overcome the local Minima of the energy function. This is accomplished by adopting a simulated annealing schedule of implementing the search of global minimum [11].

The energy function of a feedback neural network of Hopfield type is [9]

$$E = -1/2\sum_{i \neq j} w_{ij} s_i s_j, \tag{2.1}$$

where w_{ij} are the weights and s_i and s_j are the states of the network. Assuming bipolar state for each unit, the dynamics of the network is expressed as

$$s_i(t+1) = \operatorname{sign}\left[\sum_{j \neq i} w_{ij} s_j(t)\right]. \tag{2.2}$$

The state of a neural network with stochastic units is described in terms of probability distribution. In thermal equilibrium, the probability distribution of the states follows the Boltzman law :

$$P[s_\alpha] = \frac{e_\alpha^{-E} / T}{\sum_\beta e_\beta^{-E} / T}, \tag{2.3}$$

where E_α is the energy of the network in the state s_α. The network is allowed to relax to thermal equilibrium at a given temperature T. The average value of the state vector is given by

$$\langle s \rangle = \sum s_\alpha P(s_\alpha). \tag{2.4}$$

The global minima of the energy function is searched by the method of simulated annealing which requires the computation of stationary probability at thermal equilibrium for each temperature in the annealing schedule. It is obvious that the local minima of the energy function can be escaped at higher temperatures where many states are likely to be visited As the temperature is gradually reduced, the states having lower energies will be visited more frequently. Finally at $T \to 0$, the state with the lowest energy will have the highest probability. The convergence to the global minima is guaranteed only if the temperature parameter is reduced slowly (adiabatically) starting from the high value initially [12].

We may use the mean field annealing approximation [10] to speed up the process of simulated annealing. In this method, the stochastic update of the binary units is replaced by the deterministic analog states [13]. In this approximation, we may thus replace the fluctuating activation values of each unit by the average value *i.e.* x_i is replaced by $<x_i>$;

$$\langle x_i \rangle = \left\langle \sum_j w_{ij} s_j \right\rangle = \sum_j w_{ij} \langle s_j \rangle. \tag{2.5}$$

Similarly, we may write [14]

$$\langle s_i \rangle = 1P(s_i = 1/x_i) - 1P(s_i = -1/x_i). \tag{2.6}$$

Assuming the probability of update as

$$P(s_i = 1/x_i) = \frac{1}{1+\exp(-2x_i/T)}, \tag{2.7}$$

we get

$$\langle s_i \rangle = \frac{1}{1+\exp(-2x_i/T)} - \left[1 - \frac{1}{1+\exp(-2x_i/T)}\right]$$

$$= \frac{1-\exp(-2x_i/T)}{1+\exp(-2x_i/T)} = \frac{e_i^{x/T} - e_i^{-x/T}}{e_i^{x/T} + e_i^{-x/T}}$$

$$= \tanh(x_i/T). \tag{2.8}$$

Replacing x_i by $<x_i>$ and using eq. (2.5), we may reduce eq. (2.8) into the following form :

$$\langle s_i \rangle = \tanh[1/T\langle x_i \rangle]$$

$$= \tanh\left[1/T\sum_j w_{ij}\langle s_j \rangle\right]. \tag{2.9}$$

Thus, the mean field approximation involves solving the following recurrence equations involving the average values of the state units :

$$\langle s_i(t+1) \rangle = \tanh\left[1/T \sum_{j=1,\, i=1,2,\ldots N}^{N} w_{ij}\langle s_j(t) \rangle\right]. \tag{2.10}$$

These are a set of coupled non-linear deterministic equations which may be solved starting with some arbitrary values $< s_i(0) >$ initially. Once the steady equilibrium values $< s_i >$ have been obtained, let us now lower the temperature. The next set of average states at thermal equilibrium are determined using the average state values at the previous thermal equilibrium condition as the initial values $< s_i(0) >$ in the above equation for iterative solution. Here, the computation will be much faster because the deterministic set of equations are involved in this computation.

The set of equations of the mean field approximation is a result of minimization of an effective energy defined as function of the temperature. This result is given by [15]

$$\langle s_i \rangle = \tanh\left[-1/T \frac{\partial E(s)}{\partial \langle s_i \rangle} \right], \qquad (2.11)$$

where the effective energy $E(s)$ is the expression for energy of the Hopfield model.

It is obvious in the foregoing analysis that in optimization an input pattern representing the initial values for a specific optimization problem is presented to the network and the network produces a set of variables that represents a solution to the problem. If the neural network dynamics are such that the network always seeks a sizable state when the energy function is at a minimum, then the network will automatically find a solution. The inputs to the neural network are the initial state of the neural network and the final PE values represent the parameters of a solution.

3. Supersymmetry breaking by optmization

Supersymmetry is a beautiful symmetry between *bosons* and *fermions*. It leads to the ultimate form of unification free from gauge hierarchy problem and provides a natural framework to include gravity. It consists of transformation of *fermions* to *bosons* and *vice-versa*. Starting with the pioneer work of Witten [16], it has been recognized that supersymmetry could be applied to quantum mechanics as a limiting case ($N = 1$) of field theory and the subsequent development of supersymmetric quantum mechanics led to an interesting theory [17,18] where the supersymmetric Hamiltonian H is constructed in terms of non-Hermitian supercharge operators Q and Q^+ in the following form

$$H = 1/2\{Q, Q^+\} = 1/2[QQ^+ + Q^+Q]$$

such that

$$[Q, Q^+] = 0, \; Q^2 = Q^{+2} = 0,$$

$$\text{and} \quad [H, Q] = [H, Q^+] = 0, \qquad (3.1)$$

$$[Y, Q] = -iQ; \; [Y, Q^+] = iQ^+,$$

where $Y = i/2\,[\bar{\psi}, \psi]$

with ψ and $\bar{\psi}$ as fermionic variables describing spin degree of freedom. Any state satisfying the conditions :

$$Q|B\rangle = 0$$

$$\text{and} \quad Q^+|B\rangle \neq 0 \qquad (3.2)$$

is bosonic state for which we have

$$H|B\rangle = 1/2\,QQ^+|B\rangle. \qquad (3.3)$$

The fermionic state $|F\rangle$ satisfies the conditions

$$Q^+|F\rangle = 0$$

$$\text{and} \quad Q|F\rangle \neq 0 \qquad (3.4)$$

which give

$$H|F\rangle = 1/2\,Q^+Q|F\rangle$$

Let us assume that E_n is an eigen value of H with the corresponding eigen state $|n\rangle$; then

$$1/2[QQ^+ + Q^+Q]|n\rangle = E_n|n\rangle$$

$$\text{or} \quad 1/2[Q|n\rangle_+ + Q^+|n\rangle_-] = E_n|n\rangle, \qquad (3.5)$$

where $|n\rangle_+ = Q^+|n\rangle$

and $|n\rangle_- = Q|n\rangle$.

From eq. (3.5), we get

$$E_n = 1/2[{}_+\langle n|n\rangle_+ + {}_-\langle n|n\rangle_-]$$

$$\geq 0, \qquad (3.6)$$

showing that all eigen values of H are non-negative and hence the energy in supersymmetric theory is always positive. This eigen value is zero only when

$$Q|n\rangle = Q^+|n\rangle = 0 \qquad (3.7)$$

which is necessary condition for supersymmetric ground state. The supersymmetry is spontaneously broken when the ground state energy is non-zero [as depicted in Figures 1(a) and 1(b)].

Using above relations, it may readily be demonstrated [19] that the operator Q transforms the state $|F\rangle$ into state $|B\rangle$ of the eigen energy E and the operator Q^+ transforms the state $|B\rangle$ into state $|F\rangle$ i.e.

$$Q|F\rangle = E^{1/2}|B\rangle$$

$$\text{and} \quad Q^+|F\rangle = E^{1/2}|F\rangle \qquad (3.8)$$

showing that the supersymmetry pairs the *bosonic* and *fermionic* states of all positive energy states of H. On the other hand the zero energy states are not paired in this way. With $H = Q^2$, each state annihilated by H is also annihilated by Q. These states form trivial one dimensional supersymmetric multiplets. There exists the unpaired state (the ground state) if and only if the supersymmetry is an exact symmetry of the system. Thus the ground states of zero energy preserve supersymmetry while those of positive energy breaks it spontaneously. Another measure of supersymmetry is the Witten index [20], defined as the difference in the number of zero energy bosonic and fermionic states.

$$\Delta = n_B^{(0)} - n_F^{(0)}$$

If this index is non-zero *i e*.

$$\Delta \neq 0,$$

then there exists zero energy state, and hence the ground state energy is zero leading to manifestation of supersymmetry while vanishing value of Witten index *i e*

$$\Delta = 0$$

and $n_B^{(0)} = n_f^{(0)} = 0$

imply that zero energy ground state does not exist and then supersymmetry is spontaneously broken.

The supersymmetric invariant Lagrangian contains a kinetic term, the mass term and the interaction term. But when one introduces the gauge transformation, it does not remain invariant and one has to reconstruct it in a gauge covariant manner by introducing a gauge field. Supersymmetry relates particles having different spins following different statistics and as such it demands the equivalence of *fermions* and *bosons*. Such an equivalence is not observed in nature. Moreover the requirement that supersymmetric particles can only be produced in pair implies that every *bosonic* particles must have a *fermionic* partner and *vice-versa*. This type of structure of particles is not compatible with observations. As such, the supersymmetry has to be badly broken. In supersymmetric theories, the spontaneous symmetry breaking may take place in the following three manner :

(i) Broken supersymmetry with gauge symmetry intact [as shown in Figures 1(a) and 1(b)].

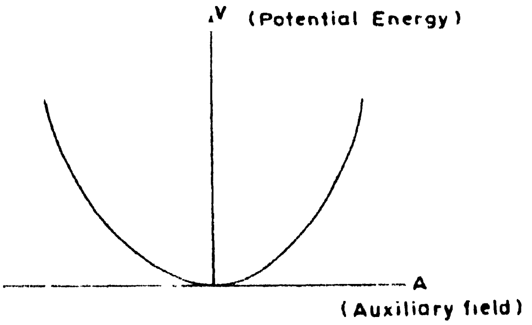

Figure 1(a). Ground state preserves the supersymmetry

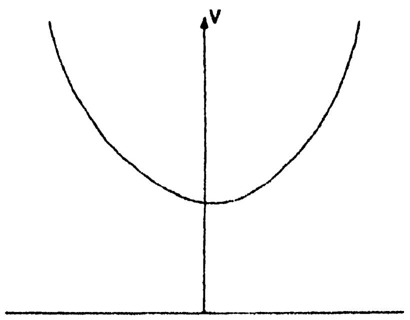

Figure 1(b). Ground state breaks the supersymmetry spontaneously

(ii) Broken gauge symmetry with supersymmetry intact (this case has been depicted in Figure 2).

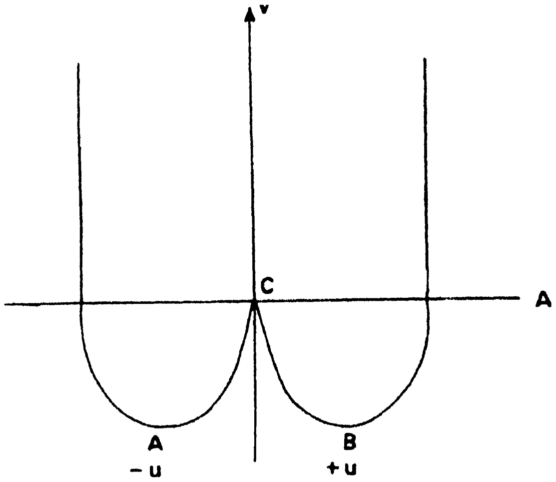

Figure 2. Only gauge-symmetry broken

(iii) Both supersymmetry and gauge symmetry broken (as depicted in Figure 3).

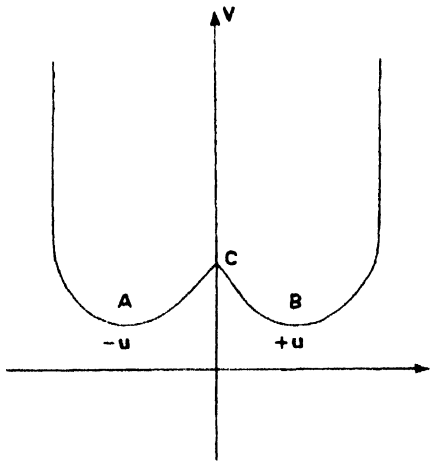

Figure 3. Both gauge-symmetry and supersymmetry are broken

The symmetry breaking depicted in Figures 2 and 3 can be done in variety of ways including the finite temperature symmetry breaking (Thermo-Field-Dynamics), where the thermal average of at least one auxiliary field does not vanish.

The spontaneous breaking of ordinary gauge symmetry depicted in Figure 2 is supersymmetric extension of Higg's mechanism where the spontaneous symmetry breaking in supersymmetric theories gives rise to a massive vector multiplet. It leads to supersymmetric generalization of Grand Unified Theories (GUT's). This mechanism of symmetry breaking also leads to supersymmetric theories [21] of monopoles and dyons. In this potential curve (Figure 2) with gauge group SU(2) we may approach the various stages of the problem using the method of optimization based on neural network principles by identifying points A and B, minima of Higg's potential, as the local minima representing the degenerate ground states ($\pm v$) of the given gauge field and the point C representing the global minimum of the supersymmetry, at which the supersymmetry is preserved. But the point C of the potential curve is extremely unstable minimum while points A and B, being the degenerate bottom of potential curve, are the stable local minima to any one of which the system will try to reach to maintain stable equilibrium. Choice of one of these ground states (A or B) by the system will break the gauge symmetry spontaneously. If we impose a physical constraint which may maintain the system at the global minimum C escaping the local minima A and B (which may be done by starting from higher temperature and reducing it gradually) then both the symmetries (gauge symmetry as well as supersymmetry) will remain maintained by the system. In this stage of optimization of the potential curve of Figure 2, at the temperature $T = 0$, the state with lowest energy corresponding to the global minimum C will have the highest possibility of maintaining both the symmetries (gauge symmetry and super symmetry) intact.

Applying this method of optimization on the potential curve of Figure 3, we may reach the different possibilities of symmetry breaking. When we start from the high value initially, the states with lower energies will be visited more frequently. As the temperature is gradually reduced and we reach at the point C, the states corresponding to ground energy (ground state) *i.e.* global minimum, will have the highest possibility. It corresponds to supersymmetry breaking leaving gauge symmetry intact because the local minima A and B have been escaped by the method of optimization using neural network. This situation corresponds to any boundary condition or environment which would distinguish between *bosons* and *fermion*. In particular, since *bosons* and *fermions* respond differently to temperature, a supersymmetric system immersed in a thermal bath would loose supersymmetry [22,23]. Let us now discuss, quantum mechanically, the effect of temperature on a supersymmetric system. In a vivid manner, supersymmetry (and in fact, any symmetry) is called broken if and only if a measurement can be made whose result depends on whether the measurement is performed before or after a symmetry transformation *i.e.* if and only if there exist some operator R whose change δR, under a symmetry transformation has a non vanishing expectation value. At zero temperature this expectation value is

$$\langle 0|\delta R|0\rangle,$$

where $|0\rangle$ is the physical ground state of the theory, whereas at finite temperature it is the thermal average

$$\langle \delta R\rangle = Z^{-1}\mathrm{tr}\left(\delta R e^{-\beta H}\right), \tag{3.9}$$

where $Z = \mathrm{tr}\, e^{-\beta H}$ is the partition function with H as Hamiltonian and $\beta = 1/kT$.

The change of R under an infinitesimal supersymmetric transformation is

$$\delta R = i\sum_{q=1}^{2}[\theta^q Q^q, R] \tag{3.10}$$

where Q^i are supersymmetric charges, and the anticommuting operators θ^1 and θ^2 are Grassmanian parameters. Then we have

$$\langle \delta R\rangle = i\sum_{q=1}\theta^q(-1)^F\langle\{Q^q, R\}\rangle \tag{3.11}$$

at $T \neq 0$ and

$$\langle 0|\delta R|0\rangle = i\sum_{q=1}^{2}\theta^q\langle 0|(-1)^F\{Q^q, R\}|0\rangle \tag{3.12}$$

for $T = 0$, where $(-1)^F$ is the Klein operator [24–26] choosing R as fermionic operator, the quantities of eqs. (3.9) and (3.10) do not vanish only for $R = Q^i$. Therefore, the unique order parameter for supersymmetric breaking are

$$\left\langle(-1)^F H\right\rangle \text{ at } T \neq 0$$

and
$$\langle 0|(-1)^F H|0\rangle = \varepsilon_0\langle 0|(-1)^F|0\rangle, \text{ at } T = 0, \tag{3.13}$$

which shows that at $T = 0$, supersymmetry is broken if and only if $\varepsilon_0 = 0$. But for a special choice of $|0\rangle$ eq. (3.11) may give zero inspite of $\varepsilon_0 \neq 0$ *i.e.*

$$|0\rangle = 1/\sqrt{2}\left[|0\rangle_+ + |0\rangle_-\right], \tag{3.14}$$

where $|0\rangle_\pm$ are orthogonal states belonging to the space of all possible vaccua (states with eigen value ε_0). These are also the eigen states of $(-1)^F$ with eigen values ± 1, *i.e.*

$$(-1)^F|0\rangle_+ = |0\rangle_+,$$

and
$$(-1)^F|0\rangle_- = -|0\rangle_-$$

Obviously, at $T \neq 0$ the relevant order parameter for supersymmetric breaking is $\langle(-1)^F H\rangle$ which is the expectation value of the charge of supersymmetric charge under a supersymmetric transformation, obtained in the conventional thermal ensemble. Other order parameters are all of the form $\langle(-1)^F R\rangle$ with R some bosonic operator. Supersymmetry is unbroken at $T \neq 0$ if and only if this order parameter vanishes.

Finite temperature supersymmetry breaking can also be discussed by applying the thermo field dynamics (TFD) [27] which is an alternative way of doing calculations in quantum mechanics. In TFD, the thermal expectation value of an operator R is represented in the thermal vacuum as

$$\langle R\rangle = \langle 0(\beta)|R|0(\beta)\rangle, \tag{3.15}$$

where $|0(\beta)\rangle$ is a finite temperature vacuum. This calculation is not possible as long as we stay within the conventional Hilbert space and hence it becomes necessary to double this

Hilbert space. If it is the Hilbert space of the theory, $|0(\beta)\rangle$ is a state in $H \otimes H$ space. *i.e.*

$$|0(\beta)\rangle = Z^{-1/2}\sum \exp(-1/2\beta E_r)|r\rangle\otimes|r\rangle, \quad (3.16)$$

where Z is the partition function.

One of the order parameter for supersymmetric breaking at finite temperature is the thermal ground state energy. Its non-vanishing value is a sure test of breakdown of supersymmetry whereas its non-vanishing value implies restoration of supersymmetry at finite temperature. Another important criteria for spontaneous breakdown of supersymmetry is that the thermal average of at least one auxiliary field does not vanish [28]. This criteria has enhanced the hope to restore the supersymmetry at finite temperature [29,30]. To analyze this stage of optimization, we start with Wess-Zumino model [31] specified by the superpotential

$$f(s) = -\lambda s + 1/2ms^2 + 1/6gs^3, \quad (3.17)$$

where s is the scalar superpotential and g is the coupling parameter. The effective potential at one loop is given by [32],

$$V^{(T)} = \sum \left|\partial f(\phi)/\partial\phi_a\right|^2 + 1/8T^2$$

$$\sum_{ab} \left|\partial^2 f(\phi)/\partial\phi_a\partial\phi_b\right|^2, \quad (3.18)$$

where $\partial\phi_a$ denotes the scalar fields. Substituting relation (3.15) into this equation, we get the following effective potential at T:

$$V^{(T)} = \left(-\lambda + m\phi + 1/2g\phi^2\right)^2 + 1/8T^2\left(m + g\phi\right)^2 \quad (3.19)$$

which is extremum (*i.e.* minimum) for

$$\phi^T = -m/g\left[1 \pm \left(1 + 2\lambda g/m^2 - g^2T^2/4m^2\right)^{1/2}\right]. \quad (3.20)$$

For T 0, $V^{(T)}$ is minimum for

$$\phi^0 = -m/g\left[1 \pm \left(1 + 2\lambda g/m^2\right)^{1/2}\right] \quad (3.21)$$

for which the minimum value of $V^{(T)}$ of eq. (3.16) comes out to be zero showing the supersymmetry is unbroken at $T = 0$.

The degeneracy of ϕ^T of eq. (3.20) can be lifted artificially at

$$T = T_C = 2/g(m^2 + 2\lambda g)^{1/2} \quad (3.22)$$

showing the occurrence of 2nd order phase transition. At this value of temperature $V^{(T)}$ is not zero and hence supersymmetry is broken, moreover supersymmetry breaking at this temperature is apparent from the non-vanishing value of $\langle F\rangle$, where F is the ordinary thermal average of the auxiliary field :

$$F = -i\partial f(s)/\partial s \quad (3.23)$$

for ϕ^T given by eq. (3.20), the following minimum value of V^T is given by eq. (3.19) :

$$V^{(T)}_{\min} = T^2/8\left[(m^2 + 2\lambda g) - g^2T^2/8\right] \quad (3.24)$$

which is non-zero in general indicating the breaking of supersymmetry at temperature T. This value tends to zero at

$$T = T^{(s)} = \sqrt{2}\,T_C = 2\sqrt{2}/g(m^2 + 2\lambda g)^{1/2} \quad (3.25)$$

which shows the restoration of supersymmetry at the temperature $T = \sqrt{2}\,T_C$. The thermal average of auxiliary field at this temperature is given by

$$\langle F\rangle = 2\lambda + m^2/g. \quad (3.26)$$

The condition for supersymmetry to remain manifest is [21]

$$\langle F\rangle = 0 \quad (3.27)$$

which is possible only if

$$\lambda = -m^2/2g$$

or $\quad \phi^0 = -m/g = \phi^{Tc}$

Under these conditions, eq. (3.25) gives

$$T^{(s)} = 0$$

indicating that supersymmetry remains manifest only at zero temperature if we take eq. (3.26) as the criterion. Thus if we rely only on the vanishing ground state energy as the sufficient test SUSY to remain manifest, we have finite temperate $T^{(s)}$ given by eq. (3.25) at which supersymmetry is restored. But as soon as we further impose the test of vanishing value of thermal average of auxiliary field to check the supersymmetry restoration, the finite temperature $T^{(s)}$ reduces to zero. This corresponds to the optimization of the curve of Figure 2.

Models of dynamic breaking of supersymmetry have been known for sometime [33] and recently, many more models have been constructed [34–36]. In all these models the classical flat directions are lifted by non-perturbation dynamics and consequently one finds run-away behavior [33]. This may not be the case if part of gauge group remains unbroken with coupling which does not tend to zero along the classical flat direction. This approach corresponds to ANN optimization of the curve of Figure 3 by adjusting the coupling (and the corresponding energy function) to establish the system at the global minimum C where supersymmetry breaks dynamically and the local minima A and B will be escaped leaving symmetry of gauge group intact.

The learning laws in ANN basically optimize certain objective function which reflects the constraints associated with a given task. Most of the learning laws utilize the gradient based approach for the optimization purpose. However, due to its deterministic nature, gradient based methods frequently get struck at local optima or saddle points. This is because the step size and step direction of the optimization process are dictated by the local information supplied by the gradient. This drawback is overcome by choosing the step direction and step size stochastically in a controlled manner. Evolutionary computation is one such method, where a population of solution are globally optimal solution. The integration of evolutionary computational technique into ANN models is Neuro-Evolutionary technique which can be used to enhance the learning capacity of the model. This technique is also useful to determine suitable topology of the network and to select proper learning rule [37].

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